Effect Of Magnetic Field On the Dispersion Of a Solute in Fluid Flow Through a Conduit With Interphase Mass Transfer

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Abstract - The combined effect of magnetic field and irreversible boundary reaction on dispersion in Newtonian fluid through a conduit (pipe/channel) is studied by using generalized dispersion model. The study explains the development of dispersive transport following the injection of a tracer in terms of three effective transport coefficients namely exchange, convective and dispersion coefficients. The absorption coefficient is seen to be independent of magnetic field. The convection coefficient is influenced by the magnetic field.

Index Terms - Generalized Dispersion model, Newtonian fluid, magnetic field, dispersion coefficient, interphase mass transfer.

1 INTRODUCTION

The study of absorption of a dispersing tracer at flow boundaries has many applications in biology, physiology, chromatography, chemical engineering, and environmental fluid mechanics. In the process of exchange of respiratory gas between body and its surroundings, the inspired air gets heated to body temperature and humidified to saturation on its way through the lung. This process and the removal of noxious gases or particulate material involve mass transfer between air stream and the walls of the respiratory tract. Gaseous dispersion takes place with chemical reaction in a wide variety of problems.

Gupta and Gupta (1972) analysed the problem of dispersion of reactive contaminants in a liquid flowing through a channel in the presence of a first order homogeneous chemical reaction using Taylor's theory for asymptotically large time and arrived at erraneous results in view of their quasi static assumptions. Sankarasubramanian and Gill (1973) investigated the phenomenon of dispersion with interphase mass transport in a Poiseuille flow through a circular tube. It was shown that the interphase mass transfer has an influence on the transport coefficients viz., exchange, convection, and diffusion coefficients. De Gance and Johns (1978a, b, 1980) extended the analysis of Sankarasubramanian and Gill (1973) to the case of a cylinder with arbitrary cross-section. They have shown that the transport coefficients were function of time and also established the full dependence of the dispersion coefficients on time, the initial solute distribution and the chemical activity of the solute.

Davidson and Schroter (1983) studied the dispersion and uptake of an inhaled slug of tissue soluble gas with in a broanchial wall as an assembly of straight rigid tubes with absorbing wall of finite thickness. Boddington and Clifford (1983) investigated the solute transport with chemical reaction, adsorption and disorption on the wall. Smith (1983) developed a delay diffusion model to study the effect of boundary absorption upon longitudinal dispersion in shear flows. The Taylor's model based on the average concentration measured on the cross-section to analyse the asymptotic dispersion process has a major set back when the tracer is chemically reactive. The removals of the chemically active solutes near the boundary moving slowly are to be left behind as an extended tail. As a consequence, skewness towards the rear in the concentration profile was noticed (Chatwin 1970, Smith 1983, Purnama 1988). This characteristic was not found through Taylor's model. Lungu and Moffat (1982) investigated the effect of wall conductance on heat diffusion using Fourier transformations. The effective diffusivity in the flow direction was a decreasing function of the wall conductance and the total rate of decrease of mass through the wall was shown to be independent of the velocity.

Taylor's method (1953) was generalized by Purnama (1988) to analyse the dispersion of a contaminant in the presence of boundary reaction. It was shown that the shear

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dispersion coefficient was increased by the presence of boundary retention. Balasubramanian *et al*. (1997) investigated the weak absorption at the boundary of a curved tube and showed that the dispersion mechanism was enhanced due to the weak losses at the flow boundary and the effect of secondary flows was to create a considerable drop in the dispersion coefficients corresponding to the case of straight tube. In their subsequent study (1999, 2003) they analysed the effects of secondary flows and boundary reaction on the dispersion using the generalized dispersion model of Sankarasubramanian and Gill (1973).The phenomenon of axial dispersion in pulsatile flow in a pipe with boundary absorption was studied numerically by Mazumdar and Das (1992). They showed that the mixing of the cross-sectionally integrated concentration of contaminant molecules was effected by the heterogeneous wall reaction. It was also observed that the dispersion coefficient asymptotically reached stationary state after a certain time and it decreased with absorption parameter.

Phillips *et al.* (1995) investigated the transport of a tracer substance through a wall layer consisting of tube containing flowing fluid surrounded by a wall layer in which the tracer was soluble. They showed that effective convection and dispersion coefficients based on the spatial moments were of little use in predicting the time-varying concentration at a fixed position as the spatial concentration profile became Gaussian only over the larger time scale i.e., when the tracer molecules took much longer time to diffuse across the wall layer than across the interior of the tube. Jayaraman *et al.* (1998) extended the model of Davidson and Schroter (1983) for the dispersion of solute in a fluid flowing through a curved tube with absorbing walls by using a mathematical model of an infinitely long conduit defined by two concentric curved circular pipes. Their results based on perturbation and spectral methods confirmed the earlier experimental findings that the influence of secondary flows on the dispersion was reduced if the tracer was very soluble in the wall. Sarkar and Jayaraman (2002) studied the effect of irreversible boundary reaction on the dispersion of tracers in annular flow using the generalized dispersion model. It was observed that the exchange and convection coefficient are enhanced while the dispersion coefficient is reduced with increase in the value of the absorption parameter. In their subsequent paper (2004) they studied the problem of dispersion with boundary absorption in an oscillatory annular flow. It was observed that the oscillatory flow arguments, the mass transfer and that an increase in the frequency parameter helps in the longitudinal dispersion of a solute.

The dispersion with interphase mass transfer in non-Newtonian fluids was also investigated by a few authors. Siddeswar and Markande (1999) analysed the unsteady convective diffusion of a solute in a micropolar fluid in a circular pipe. Nagarani *et al.* (2004) studied the effect of non-Newtonian rheology on the dispersion of a solute in a Casson fluid with interphase mass transfer at the boundary using the generalized dispersion model. In a subsequent paper (2008) they extended their study to the effect of boundary absorption to Casson fluid in an annulus. Ramana (2011) studied the analysis of Nagarani and Sarojamma (2004, 2008) to Herschel-Bulkley fluid.

Here we make an attempt to analyse the effect of magnetic field on the dispersion of a solute in a conduit with interphase mass transfer at the outer boundary. Section 2 gives the mathematical formulation of the problem in pipe flow analysis with appropriate initial and boundary conditions. Section 3 presents the channel flow analysis. In section 4 the results are discussed on the effect of wall absorption parameter and magnetic field on the three dispersion coefficients viz., exchange, convection and dispersion coefficients and mean concentration. Conclusions are presented in section 5.

2 PIPE FLOW ANALYSIS

2.1 Mathematical Formulation

Let us consider the dispersion of a bolus of solute that is initially distributed in a circular tube of radius ‘a’. The flow in the tube is considered to be axi- symmetric fully developed, steady, laminar and the fluid is Newtonian fluid. The non dimensional unsteady convective diffusion equation which describes the local concentration C of the solute as a function of axial coordinate z, radial coordinate r and time t can be written in the form

$$\frac{\partial C}{\partial t} + w\frac{\partial C}{\partial z} = \left(L^2 + \frac{1}{Pe^2}\frac{\partial^2}{\partial z^2}\right)C \qquad (1)$$

with the non-dimensional variables

$$C = \frac{\bar{C}}{C_0},\ w = \frac{\bar{w}}{w_0},\quad r = \frac{\bar{r}}{a},\quad z = \frac{D_m \bar{z}}{a^2 w_0},$$

$$t = \frac{D_m \bar{t}}{a^2} \qquad (2)$$

where $L^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)$, t is the non-dimensional time, C_0 is the reference concentration, w is the non-dimensional axial velocity of the fluid in the axial direction z ,

$w_0 = -\frac{a^2}{4\mu_\infty}\frac{d\bar{p}}{d\bar{z}}$ is the characteristic velocity, μ_∞ is the viscosity of the fluid, $\frac{d\bar{p}}{d\bar{z}}$ is the applied pressure gradient along the axis of the pipe, D_m is coefficient of molecular diffusion (molecular diffusivity) which is assumed to be constant and $Pe = \frac{aw_0}{D_m}$, Peclet number. The variables with bars represent the corresponding dimensional quantities.

Initial and Boundary Conditions

2.2 Initial Conditions

At an instant of time, the amount of tracer left in the system, its convective velocity, and the extent of shear distribution depend upon the initial discharge. Following Sankarasubramanian and Gill (1973), we consider the initial distribution at t = 0 as the case when the solute of mass m is introduced instantaneously at the plane z = 0 uniformly over the cross section of a circle of radius d (where $0 < d \leq 1$) concentric with the tube. Hence, in terms of non-dimensional quantities, the initial distribution assumed to be in a variable separable form is given by

$$C(0, z, r) = \psi(z) Y(r) \qquad (3)$$

with $\psi(z) = \frac{\delta(z)}{d^2 Pe}$ (4a)

and $Y(r) = 1, 0 < r < d$ (4b)

$= 0, d < r < 1$

where $\delta(z)$ is the Dirac delta function.

2.3 Boundary conditions

The general boundary condition at the wall can be written as

$$\frac{\partial C}{\partial r} = -\beta C + \beta_1 \qquad (5a)$$

This equation can be written in different forms by selecting the constants appropriately ,

(i) Heterogeneous Chemical reaction of the first order ($\beta_1 = 0$)

(ii) Non-equilibrium interphase ($\beta_1 = \beta C_B$, where C_B is the surface concentration)

(iii) Constant flux across the wall ($\beta = 0$)

Boddington and Clifford (1983) considered the dispersion of a very reactive in a gas flowing in a circular quartz tube along with the irreversible loss of the reactive species of concentration C by heterogeneous reaction on the wall. In this case the boundary condition at the tube wall is

$$\frac{\partial C}{\partial r} = -\beta C + \beta_2 \frac{\partial C_B}{\partial t} \qquad (5b)$$

where c_B is the surface concentration satisfying

$$\beta_2 \frac{\partial C_B}{\partial t} = \beta_3 C - \beta_4 C_B \qquad (5c)$$

where β_2 is the ratio of the amount of material absorbed to that in the fluid phase. The constants β , β_3, β_4 respectively, account for the irreversible loss of absorption and desorption at the flow boundary. Jayaraman *et al.* (1998) studied the pattern of the dispersion and uptake of an inhaled slug of tissue soluble gas in a curved tube using a two phase model, with the appropriate boundary condition at the interface as

$$\beta_5 \frac{\partial C_1^1}{\partial r} = \beta_6 \frac{\partial C_2^1}{\partial r} \qquad (5d)$$

where C_1^1 and C_2^1 are the solute concentrations and β_5 and β_6 are the molecular diffusion coefficients in the two phases of the model. At the interface a linear equilibrium relation was introduced so that

$$C_2^1 = \beta_7 C_1^1 \qquad (5e)$$

where β_7 is the solubility coefficient.

In the present model, we consider the boundary conditions

$$\frac{\partial C}{\partial r}(t, z, 1) = -\beta C(t, z, 1) \qquad (6)$$

where β is the non-dimensional wall absorption parameter.

As the amount of solute in the system is finite

$$C(t, \infty, r) = \frac{\partial C}{\partial z}(t, \infty, r) = 0 \qquad (7)$$

and $C(t, z, 0)$ = finite (8)

The velocity distribution for an axi-symmetric, fully developed, steady, laminar flow of a Newtonian fluid in a circular pipe under the influence of a transverse magnetic field in dimensionless form is

$$w = \frac{P}{M^2}\left[1 - \frac{I_o(M r)}{I_o(M)}\right] \qquad (9)$$

2.4 Method of Solution

The convective diffusion equation (1) along with the initial and boundary conditions (3) and (6 - 8) is solved using the derivative expansion method developed by Sankarasubramanian and Gill (1973).

$$C = \sum_{n=0}^{\infty} f_n(t, r) \frac{\partial^n C_m}{\partial z^n} \qquad (10\ a)$$

where the average concentration C_m is defined as

$$C_m = 2\int_0^1 C\, r\, dr \qquad (10\ b)$$

Multiplying equation (1) by *2r* and integrating with respect to *r* from 0 to 1, we obtain

$$\frac{\partial C_m}{\partial t} = \frac{1}{Pe^2}\frac{\partial^2 C_m}{\partial z^2} + 2\frac{\partial C}{\partial r}(t,z,1) - 2\frac{\partial}{\partial z}\int_0^1 w(t,r)\, C(t,z,r)\, r\, dr$$

(11)

Introducing (10 a) into (11), the following dispersion model for C_m is obtained as

$$\frac{\partial C_m}{\partial t} = \sum_{n=0}^{\infty} K_n(t)\frac{\partial^n C_m}{\partial z^n} \qquad (12)$$

where K_n's are given by

$$K_n(t) = \frac{\delta_{n,2}}{Pe^2} + 2\frac{\partial f_n}{\partial r}(t,1) - 2\int_0^1 f_{n-1}(t,r)\, w(t,r)\, r\, dr$$

n = 0, 1, 2,… , $f_{-1} = 0$

(13)

$\delta_{n,2}$ denotes the Kronecker delta.

The equation (12) can be truncated after the term involving K_2.Thus the distribution of mean concentration C_m , can be described by the generalized dispersion model as

$$\frac{\partial C_m}{\partial t} = K_0(t)C_m + K_1(t)\frac{\partial C_m}{\partial z} + K_2(t)\frac{\partial^2 C_m}{\partial z^2} \qquad (14)$$

The absorption parameter $K_0(t)$ arises due to the nonzero solute flux at the tube wall. This will be negative in this problem to account for the depletion of solute in the system with time caused by the irreversible reaction occurring at the tube wall. If the solute were to be generated at the wall according to first-order process, β in equation (6) would assume negative sign and then the exchange coefficient $K_0(t)$ would be positive. $K_1(t)$ and $K_2(t)$ correspond to the convective and dispersion coefficients, respectively. The convection coefficient $K_1(t)$ accounts for the velocity of the reactive tracer and the dispersion coefficient $K_2(t)$ provides the modifications in the convective dispersion, occurring owing to absorption.

Substituting (10a) in equation (1), using (12) and equating the coefficients of $\frac{\partial^n C_m}{\partial z^n}$, *n* = 0, 1, 2, ------, gives the following set of partial differential equations for f_n as

$$\frac{\partial f_n}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f_n}{\partial r}\right) - w(t,r)f_{n-1} + \frac{1}{Pe^2}f_{n-2} - \sum_{i=0}^{n} K_i f_{n-i}$$

, *n* = 0, 1, 2, ….. (15)

where $f_{-1} = f_{-2} = 0$

From equation (4), (6) to (8) we obtain the initial and boundary conditions on C_m and f_n as follows

$$C_m(0, z) = 2\psi(z)\int_0^1 Y(r)\, r\, dr \qquad (16a)$$

gives $f_0(0, r) = \dfrac{Y(r)}{2\int_0^1 Y(r)\, r dr}$

(16b) by setting $f_n(0, r) = 0$, *n* = 1, 2….. (16c)

From equation (6) to (8), the boundary conditions are

$$\frac{\partial f_n}{\partial r}(t,1) = -\beta f_n(t,1), \quad n = 0,1,2,\ldots.$$

(16d)

$f_n(t, 0)$ = finite *n* = 0,1, 2,… (16e)

$$C_m(t,\infty) = \frac{\partial C_m}{\partial z}(t,\infty) = 0$$

(16f)

$$\int_0^1 f_n(t,r)\, r\, dr = \frac{1}{2}\delta_{n0} \quad \text{for } n = 0, 1, 2\ldots.$$

(16g) Since the equations (13) and (15) are coupled in order to determine the dispersion coefficient $K_2(t)$ we need to obtain the pair of functions (f_n, K_n), *n* = 0,1,…., one after the other.

Evaluation of f_0 (t, r) and K_0(t)

The function f_0 and exchange coefficient K_0 are independent of velocity field and can be solved immediately. From (13) we have

$$K_0(t) = 2\frac{\partial f_0}{\partial t}(t, 1) \qquad (17)$$

To determine $K_0(t)$ we have to first evaluate f_0 *(t , r)*. Thus the equation for f_0 may be written from equation (15) as

$$\frac{\partial f_0}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f_0}{\partial r}\right) - f_0 K_0 \qquad (18)$$

The initial and boundary conditions on f_0 are shown in equations (16 b, d, e) .

From (16g) we have

$$\int_0^1 f_0(t,r)\, r\, dr = \frac{1}{2} \qquad (19)$$

Using (17), the solution of $f_0(t, r)$ satisfying the initial and boundary conditions is given by

$$f_0(t, r) = \frac{\sum_0^\infty A_n J_0(\mu_n r) e^{-\mu_n^2 t}}{2\sum_0^\infty (\frac{A_n}{\mu_n}) J_1(\mu_n) e^{-\mu_n^2 t}} \quad (20)$$

where $A_n = \frac{\mu_n^2 \int_0^1 r\, Y(r)\, J_0(\mu_n r)\, dr}{(\mu_n^2+\beta^2) J_0^2(\mu_n) \int_0^1 r Y(r)\, dr}$ (n = 0, 1, 2……..)

(21a)

μ_n 's are the roots satisfying the transcendental equation

$$\mu_n J_1(\mu_n) = \beta J_0(\mu_n) \quad (n = 0, 1, 2……..)$$

(21b)

where J_0, J_1 are Bessel functions of orders zero and one, respectively.

From equation (17) and (20) the exchange coefficient can be written as

$$K_0(t) = -\frac{\sum_0^\infty A_n \mu_n J_1(\mu_n) \exp(-\mu_n^2 t)}{\sum_0^\infty (\frac{A_n}{\mu_n}) J_1(\mu_n) \exp(-\mu_n^2 t)} \quad (22)$$

which is exactly the same as derived by Sankarasubramanian and Gill (1973).

Asymptotic Expansions for f_n's and K_n's for n = 0, 1, 2, …… for steady flow

It is necessary to determine the remaining functions f_n introduced in equation (15) in order to generate the dispersion coefficient. However, owing to the coupling effect between $f_n(t, r)$ and $K_n(t)$, the calculation of higher order dispersion coefficients and time dependent parts of the dispersion coefficients become tedious. Therefore, we will content ourselves with the asymptotic steady-state representations of $f_n(t, r)$ and $K_n(t)$ for the case of steady flow, since these asymptotic values provide useful physical insight into the behaviour of the system. Hence, we will obtain solutions (f_n, K_n), n = 0, 1, 2…… for large times, so that the dispersion model defined in (15) is a representation of the asymptotic results under steady state conditions.

As $t \to \infty$, equations (20) and (22) give the following asymptotic representation for f_0 and K_0 :

$$f_0(\infty, r) = \frac{\mu_0}{2J_1(\mu_0)} J_0(\mu_0 r) \quad (23)$$

$$K_0(\infty) = -\mu_0^2 \quad (24)$$

where μ_0 is the first root of equation (21b) with least magnitude.

For large values of time, the steady state function $f_n(r)$ satisfies the equation

$$\frac{1}{r}\frac{d}{dr}\left(r \frac{d f_n}{dr} \right) + \mu_0^2 f_n = w(r) f_{n-1} - \frac{1}{(Pe)^2} f_{n-2} + \sum_{i=1}^n k_i f_{n-i}$$

where n =1, 2, 3,……

(25)

The boundary conditions on $f_n(r)$ are

$f_n(0)$ = finite $\quad f_n'(1) = -\beta f_n(1)$ n = 1, 2, 3,……

(26)

$$\int_0^1 f_n r\, dr = 0 \quad (n = 1, 2, 3, ...) \quad (27)$$

For large times the equation (11) for K_n's reduces to

$$K_n = \frac{\delta_{n2}}{(Pe)^2} + 2 f_n'(1) - 2\int_0^1 w(r) f_{n-1}(r)\, dr \quad (28)$$

(n =1, 2, 3…).The use of the solvability condition in equation (27) gives the expression for K_n as

$$K_n = \frac{\int_0^1 r J_0(\mu_0 r) \left\{ \frac{1}{Pe^2} f_{n-2}(r) - w(r) f_{n-1}(r) - \sum_{i=1}^{n-1} K_i f_{n-i}(r) \right\} dr}{\int_0^1 r f_0(r) J_0(\mu_0 r) dr}$$

$$(n = 1, 2, 3, ...) \quad (29)$$

where $\sum_{i=1}^{n-1}$ applies for n ≥ 2.

For n =1, the expression for the asymptotic convective coefficient K_1 can be obtained as

$$K_1 = \frac{-\int_0^1 w(r) f_0(r)\, r J_0(\mu_0 r)\, dr}{\int_0^1 f_0(r) r J_0(\mu_0 r)\, dr} \quad (30a)$$

$$= \frac{-p\mu_0^2}{(\mu_0^2+\beta^2) M^2} \left\{ 1 + \frac{J_1^2(\mu_0)}{J_0^2(\mu_0)} - \frac{2}{J_0^2(\mu_0) I_0(M)} \int_0^1 \left[r I_0(M r) J_0^2(\mu_0 r) \right] dr \right\} \quad (30b)$$

For large times, the differential equation for f_1 from equation (25) can be written as

$$\frac{1}{r}\frac{d}{dr}\left(r \frac{df_1}{dr}\right) + \mu_0^2 f_1 = w(r) f_0 + K_1 f_0 \quad (31)$$

The boundary conditions for f_1 are

$f_1(0)$ = finite, $\quad f_1'(1) = -\beta f_1(1)$ (32)

and $\int_0^1 f_1 r\, dr = 0$ (33)

The solution for f_1 satisfying the corresponding boundary conditions (32) and (33) is obtained using equations (30) and (31) as

$$f_1(r) = \sum_{n=0}^{\infty} B_n J_0(\mu_n r) \qquad (34)$$

From equation (34), with the help of the condition (33) we get

$$B_0 = \frac{-\mu_0}{J_1(\mu_0)} \sum_{n=1}^{\infty} B_n \frac{J_1(\mu_n)}{\mu_n} \qquad (35)$$

Using equations (34) and (35), the expression for f_1 can be written as

$$f_1 = \sum_{n=1}^{\infty} B_n \left[J_0(\mu_n r) - \frac{\mu_0}{J_1(\mu_0)} \frac{J_1(\mu_n)}{\mu_n} J_0(\mu_0 r) \right] \qquad (36)$$

where B_n 's are given by

$$B_n = \frac{2\mu_n^2}{(\mu_n^2 - \mu_0^2)(\mu_n^2 + \beta^2) J_0^2(\mu_n)} \int_0^1 [w(r) + k_1] r f_0(r) J_0(\mu_n r) dr \qquad (37)$$

Using (29), (36) and (37) the dispersion coefficient is obtained as

$$K_2 = \frac{1}{Pe^2} + \frac{4\mu_0 J_1(\mu_0)}{(\mu_0^2 + \beta^2) J_0^2(\mu_0)} \frac{p}{M^2 I_0(M)} \sum_{n=1}^{\infty} B_n \int_0^1 [r J_0(\mu_n r) J_0(\mu_0 r) I_0(M r)] dr \qquad (38)$$

Solution for Mean Concentration

The mean concentration C_m is obtained from equation (14) with initial and boundary conditions given by (16a) and (16f) and is given by

$$C_m(t,z) = \frac{1}{2(Pe)\sqrt{\pi \xi}} \exp\left(\zeta - \frac{z_1^2}{4\xi}\right) \qquad (39)$$

where $\zeta(t) = \int_0^t K_0(\eta)\, d\eta$ (40)

$$z_1(t,z) = z + \int_0^t K_1(\eta)\, d\eta \qquad (41)$$

$$\xi(t) = \int_0^t K_2(\eta)\, d\eta \qquad (42)$$

4 CHANNEL FLOW ANALYSIS

Mathematical Formulation

Let us consider the Cartesian co-ordinate system ($\bar{x}, \bar{z}$), where $\bar{x}$ denotes the transverse co-ordinate and $\bar{z}$ denotes the axial co-ordinate to describe the dispersion of a solute in a Newtonian fluid flowing in a channel. Assume that the flow is steady, fully developed, laminar and axi-symmetric. The non-dimensional form of the unsteady convective diffusion equation for the dispersion of the solute in the channel flow, is given as

$$\frac{\partial C}{\partial t} + w \frac{\partial c}{\partial z} = \left(\nabla^2 + \frac{1}{Pe^2} \frac{\partial^2}{\partial z^2} \right) C \qquad (43)$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2}$ and $Pe = \frac{a w_0}{D_m}$ (Peclet number) (44)

'*a*' is half of channel width, and w_0 is the characteristic velocity given by

$$w_0 = -\frac{a^2}{2\mu_\infty} \frac{d\bar{p}}{d\bar{z}} \qquad (45)$$

The initial and boundary conditions in the dimensionless form are

$$C(0, z, x) = \psi(z) X(x) \qquad (46a)$$

$$\frac{\partial C}{\partial r}(t, z, 1) = -\beta C(t, z, 1) \qquad (46b)$$

$$C(t, \infty, x) = \frac{\partial C}{\partial z}(t, \infty, x) = 0 \qquad (46c)$$

$$\frac{\partial C}{\partial x}(t, z, 0) = 0 \qquad (46d)$$

The velocity distribution for an axi-symmetric, fully developed, steady, laminar flow of a Newtonian fluid in a channel, in non-dimensional form, is obtained as (3.38)

$$w = \frac{P}{M^2} \left[1 - \frac{\cosh M x}{\cosh M} \right] \qquad (47)$$

Method of Solution

The unsteady convective diffusion equation (43) has to be solved for the local concentration C subject to the initial and boundary conditions (46) with axial velocity w given by equation (47). In the present analysis the mean concentration is defined as

$$C_m = \int_0^1 C\, dx \qquad (48)$$

Following the same procedure as in the case of pipe flow analysis to find the solution for the problem, the expressions for K_n's and f_n's are given by

$$\frac{\partial f_n}{\partial t} = \frac{\partial^2 f_n}{\partial x^2} - w(r) f_{n-1} + \frac{1}{Pe^2} f_{n-2} - \sum_{i=0}^{n} K_i f_{n-i} \qquad (49)$$

where $n = 0, 1, 2\ldots$

$$K_n = \frac{\int_0^1 \cos(\mu_0 x)\left\{\frac{1}{Pe^2} f_{n-2}(x) - w(x) f_{n-1}(x) - \sum_{i=1}^{n-1} K_i f_{n-i}(r)\right\} dx}{\int_0^1 f_0(x)\cos(\mu_0 x)dx}$$

, n = 1, 2 …… (50)

and $K_0 = \frac{\partial f_0(t,1)}{\partial x}$ (51)

The initial and boundary conditions are given as

$$f_0(0, x) = \frac{X(x)}{2\int_0^1 X(x)\,dx} \quad (52a)$$

$f_n(0, x) = 0$, $n = 0,1,2\ldots..$ (52b)

$$\frac{\partial f_n}{\partial x}(t,1) = -\beta\, f_n(t,1), \; n = 0,1,2,\ldots.$$ (52c)

$$\frac{\partial f_n}{\partial x}(t,0) = 0, \; n = 0,1,2,\ldots$$ (52d)

$$C_m(t, \infty) = \frac{\partial C_m}{\partial z}(t, \infty) = 0$$ (52e)

$$\int_0^1 f_n(t,x)\; dx = \delta_{n0}$$ (52f)

Due to the change in the definition of C_m and the operator ∇^2, some minor modifications in the solvability condition and the expression for K_n, n = 0, 1, 2…. . are obtained. With out going into details again, the solutions for K_0, K_1, K_2 and f_0, f_1 are written as given below:

$$f_0(x,t) = \frac{\sum_0^\infty A_n \cos(\mu_n x)\; e^{-\mu_n^2 t}}{\sum_0^\infty (A_n/\mu_n)\sin(\mu_n)\; e^{-\mu_n^2 t}} \quad (53)$$

$$K_0(t) = \frac{-\sum_0^\infty A_n \mu_n \sin(\mu_n)\; e^{-\mu_n^2 t}}{\sum_0^\infty (A_n/\mu_n)\sin(\mu_n)\; e^{-\mu_n^2 t}} \quad (54)$$

where $A_n = \frac{2\mu_n^2}{\mu_n^2 + \beta\cos^2(\mu_n)} \frac{\int_0^1 X(x)\cos(\mu_n x)\,dx}{\int_0^1 X(x)\,dx}$ (55)

and μ_n's are the roots of the equation

$$\mu_n \sin(\mu_n) = \beta \cos(\mu_n)$$ (56)

The absorption, convection and dispersion coefficients are obtained for large times as in the case of pipe flow analysis. Hence, the asymptotic expansions for f_n's and K_n's for n = 0, 1, 2 in channel flow analysis are obtained as

$$f_0(\infty, x) = \frac{\mu_0 \cos(\mu_0 x)}{\sin\mu_0}$$ (57)

$$K_0(\infty) = -\mu_0^2 \quad (\mu_1 > \mu_0)$$ (58)

$$K_1 = -\frac{\int_0^1 w(x)\; f_0(x)\cos(\mu_0 x)dx}{\int_0^1 f_0(x)\cos(\mu_0 x)dx} \quad (59a)$$

$$= -\frac{p}{M^2}\{1 - C_1 * [C_2 + C_3]\}$$ (59b)

where

$$C_1 = \frac{2\,\mu_0^2}{\cosh M\;(\mu_0^2 + \beta\cos^2\mu_0)}$$ (59c)

$$C_2 = \frac{2\,\beta\mu_0\cosh(M)\;\cos^2\mu_0}{\mu_0\,(4\mu_0^2 + M^2)} + \frac{\sinh M}{2M}$$ (59d)

$$C_3 = \frac{M\cos(2\,\mu_0)\sinh M}{2\,(4\mu_0^2 + M^2)}$$ (59e)

$$f_1 = \sum_1^\infty B_n [\cos(\mu_n x) - \frac{\mu_0}{\sin\mu_0}\frac{\sin\mu_n}{\mu_n}\cos(\mu_0 x)]$$ (60)

where B_n's are given by

$$B_n = \frac{2\mu_n^2}{(\mu_0^2 - \mu_n^2)(\mu_n^2 + \beta\cos^2\mu_n)} \times \int_0^1 (w(x) + K_1) f_0(x)\cos(\mu_n x)\,dx$$ (61)

$$K_2 = \frac{1}{pe^2} - \frac{2\beta\cos\mu_0}{(\mu_n^2 + \beta\cos^2\mu_n)}\int_0^1 (w(x) + K_1) f_1(x)\cos(\mu_0 x)\,dx$$ (62)

5. RESULTS AND DISCUSSION

The objective of the present analysis is to study the effect of magnetic field on the dispersion process following the injection of a chemically active tracer in a solvent flowing through a tube/channel with reactive boundary. Integrals involved in solving the transport coefficients are evaluated numerically using Simpson's rule. It is observed that the exchange (absorption) coefficients K_0 (t) is not influenced by magnetic field but depends on wall reaction parameter. Further, the asymptotic convection coefficient K_1 and asymptotic dispersion coefficient K_2 are dependent on magnetic field as well as wall absorption parameter β. In the

present study the value of the wall absorption parameter β is taken in the range of 0.01 to 100. The range for the Hartmann number M is taken as 1 - 3

Asymptotic Exchange Coefficient K_0

The variation of $-K_0$ versus the absorption parameter β for large times is described in Fig 1 (a, b). The magnitude of the exchange coefficient K_0 increases steadily with β and attains 5.7 as β assumes very large value that is 100. When the absorption parameter takes very large values the reaction at the wall consumes material very rapidly than it can be supplied by molecular diffusion. Thus the concentration at the wall becomes zero and the mass transport process in the system becomes diffusion controlled. The corresponding variation of K_0 versus β in channel case is presented in Fig 1(b). The qualitative behavior of $-K_0$ is similar to that of the pipe case. The values of $-K_0$ in channel are less than half of the values in the pipe flow. Thus it is inferred that the absorption of solutes at the boundary is more in pipe.

Asymptotic Convection Coefficient K_1

Figs 2(a, b) describes the variation of negative asymptotic convection coefficient $-K_1$, versus the wall absorption parameter β for different values of Hartmann number. The presence of magnetic field decreases the value of the asymptotic convection coefficient and they increase with the increase in the wall absorption parameter .The values of $-K_1$ when M = 1 are reduced five times to that of the non-magnetic case. Increase in the Hartmann number further decreases the value of $-K_1$. When M = 3 the value of $-K_1$ is observed to be half of the corresponding value of the case when M = 1. The reason for the reduction is to deplete the solute in the slower moving wall region and the solute is weighed in the faster moving central region (in non-magnetic case) . Therefore, the solute is convected along at a velocity higher than the average flow velocity. The reduction in $-K_1$ with increase in M is due to the reduction in the corresponding velocities. The values of $-K_1$ in channel flow analysis are higher than those of the pipe flow case.

Asymptotic dispersion coefficient K_2

The asymptotic dispersion coefficient K_2 (from which the additive contribution of the axial diffusion $\frac{1}{Pe^2}$ is deducted) in pipe and channel as a function of β for different values of M is presented in Figs 3(a, b). It is observed that the axial dispersion is significantly decreased by the boundary reaction. When M = 1and β = 100 in pipe (channel) the dispersion coefficient is reduced by four (seven) times of the value corresponding to that of the case β = 0.01. The corresponding reduction factor in the absence of magnetic field is also four (seven) times. This might be due to the smaller velocity gradients in the central region of the pipe than near the wall and larger velocity gradients across the solute distribution gives larger axial dispersion and hence the axial dispersion is decreased. Also, transverse diffusion inhibits axial dispersion and larger transverse concentration gradients in the system with wall reaction tend to increase transverse diffusion and therefore decrease the axial dispersion coefficient. The dispersion coefficients reduce predominantly with increase in the Hartmann number M. When M = 1 depending upon the wall reaction parameter say (0.01 to 100) the reduction factor in the dispersion coefficient is in the range (22.76 - 23.32) in comparison to the values of the non magnetic case. The corresponding reduction in channel varies in the range (8.33 - 8.78). The values of K_2 in the limiting case as M→0 correspond to the results of Sankarasubramanian and Gill (1973). Figs 4(a, b) describes the variation of K_2 verses Hartmann number. It is noticed that the dispersion coefficient in pipe (channel) flow analysis decreases with increase in Hartmann number and as M approaches 5 (4), K_2 approaches the value zero. In this case flow becomes more plug like and the dispersion disappears.

Mean Concentration

The variation of mean concentration C_m as a function of the non-dimensional time for different values of the Hartmann number at z = 0.5 is plotted in pipe (channel) in Fig 5(a, b). The profiles of the mean concentration are obtained from the solution of equation (39).

The mass transport coefficients in the expression for C_m are approximated by the corresponding asymptotic values. The mean concentration C_m reduces with time due to the constant depletion occurring at the boundary of the pipe. From 5(a), when M = 1, β = 0.01and z = 0.5 the peak value of mean concentration in pipe (channel) is 7.88 (6.07) while in the absence of magnetic field it is 3.85(3.54). For small values of absorption parameter i.e., β = 0.01 in the absence of magnetic field the peak value of mean concentration is attained faster (t = 0.981) than that in the presence of magnetic field (t = 4.63, M = 1). In channel case they are attained much faster i.e., at t = 0.73 in the absence of magnetic field it is at t = 2.08 when M = 1. The peak value of mean concentration in pipe flow (channel) in the presence of magnetic field is

twice (almost twice) that of the value corresponding to that of the non-magnetic case.

From Fig 6(a, b) for higher values of absorption parameter that is $\beta = 1.0$, the behavior is almost reversed i.e., in the absence of magnetic field the peak value of concentration is obtained faster than that of the corresponding case in the presence of magnetic field in both pipe and channel flow analyses. The peak value of mean concentration in pipe (channel) flow analysis in the absence of magnetic field is 1.144 (2.641) while in the presence of magnetic field M = 1 it is 0.017 (1.861).

6. CONCLUSIONS

The effect of magnetic field on the dispersion of a solute in a fluid flow with boundary retention effects in a conduit is discussed using the generalized dispersion model of Sankarasubramanian and Gill (1973).The dispersion process is described through the three transport coefficients i.e., exchange (absorption) coefficient, convection coefficient and dispersion coefficient. The absorption coefficient is seen to be independent of magnetic field. The convection coefficient is influenced by the magnetic field. It is observed that the negative asymptotic convection coefficient decreases with increase in magnetic field and increases with increase in the wall absorption parameter. The values of $-K_1$ when M = 1 are reduced five times that of the non-magnetic case. Increase in the Hartmann number further decreases the value of $-K_1$.The reduction in $-K_1$ with increase in M is due to the reduction in the corresponding velocities owing to the Lorentz force.

The axial dispersion is significantly decreased by the boundary reaction. When M = 1 and β =100 in pipe (channel), the dispersion coefficient is reduced by four (seven) times of the value corresponding to that of the case β = 0.01.The mean concentration C_m reduces with time due to the constant depletion taking place at the boundary. When M = 1, $\beta = 0.01$ and z = 0.5, the peak value of mean concentration in pipe (channel) is 7.88 (6.07) while in the absence of magnetic field it is 3.85 (3.54). In the absence of magnetic field the peak value is attained faster than that in the presence of magnetic field. But for higher value of absorption parameter that is β = 1, almost a reversed behavior is observed. The peak value of mean concentration in pipe (channel) flow analysis in the absence of magnetic field is 1.44 (2.641) while in the presence of magnetic field M = 1 it is 0.017 (1.861).

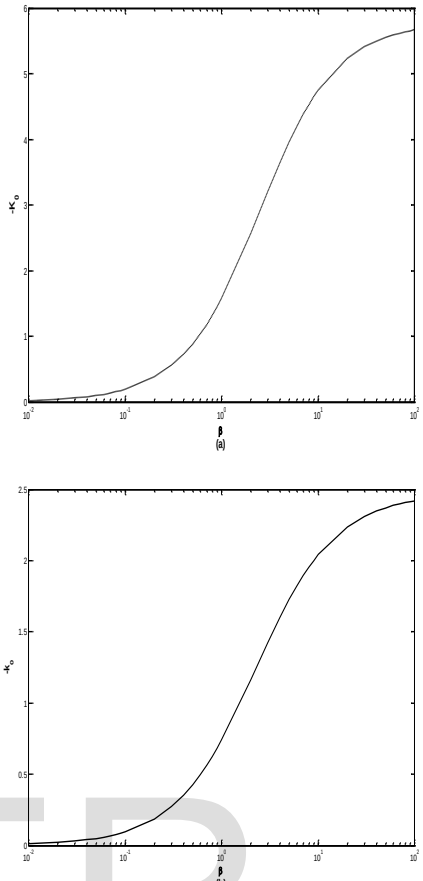
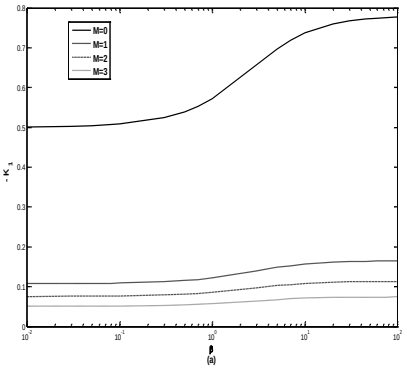


Fig 1Variation of negative asymptotic absorption coefficient $-K_0$ with absorption parameter β in (a) Pipe (b) Channel



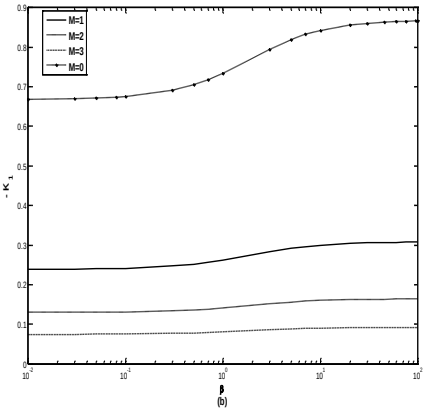


Fig 2 Variation of negative asymptotic convection coefficient $-K_1$ with absorption parameter β for different values of M in (a) Pipe (b) Channel

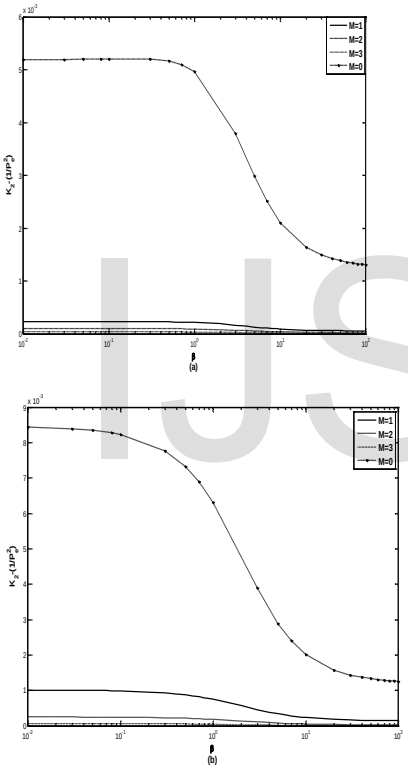


Fig 3 Variation of negative asymptotic dispersion coefficient $K_2 - (1/Pe^2)$ with absorption parameter β for different values of M in (c) Pipe (d) Channel

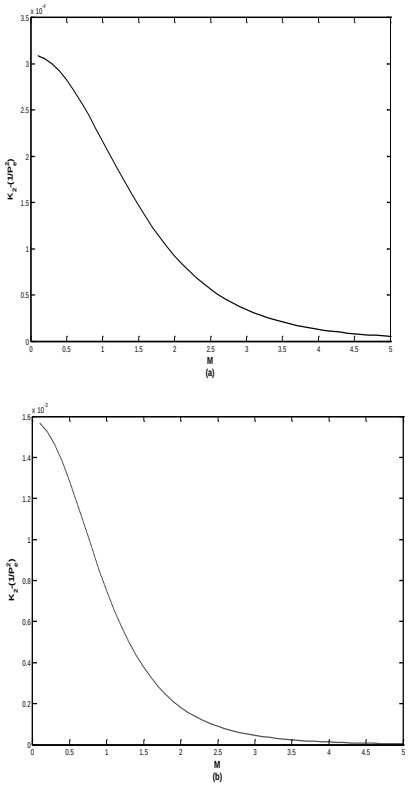


Fig 4 Variation of negative asymptotic dispersion coefficient $K_2 - (1/Pe^2)$ with Hartmann number M in (a) Pipe (b) Channel

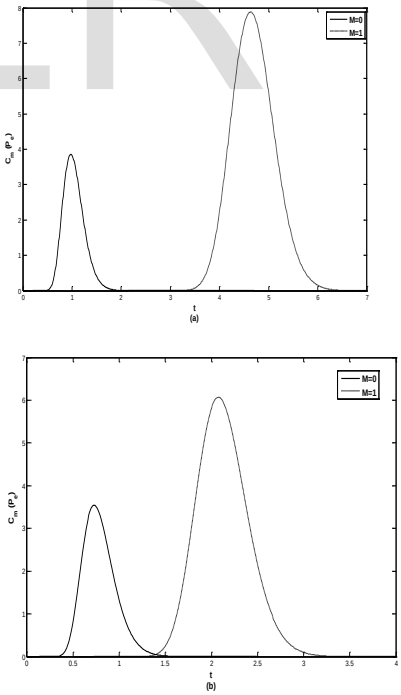


Fig 5 Variation of mean concentration C_m with time for different values of M when z=0.5 and β=0.01 (a) Pipe (b) Channel

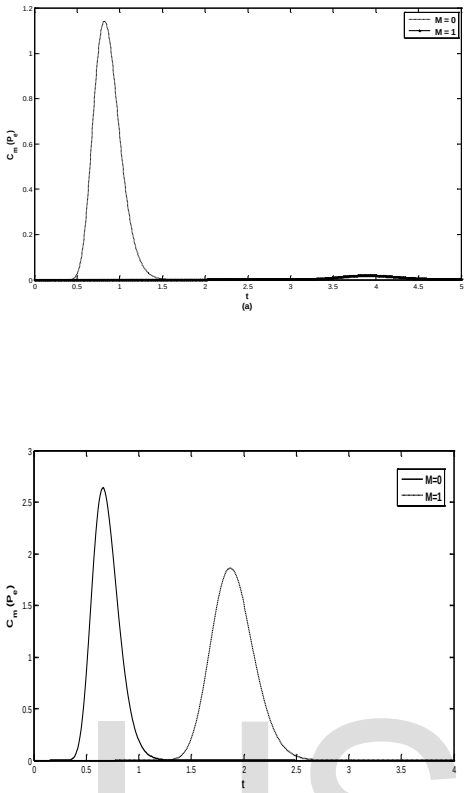


Fig 6 Variation of mean concentration C_m with time for different values of M when z=0.5 and β=1 (a) Pipe (b) Channel

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